Non-Classical Light

Quantum Optics Lecture Series

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Lecture Outline

I. Dynamics

Quadrature plane motion & Phase space evolution.

II. Generation

Squeeze operators & Nonlinear optics (PDC).

III. Detection

Homodyne systems & LIGO applications.

Dynamics in Quadrature Plane

Evolution of single frequency light ω:

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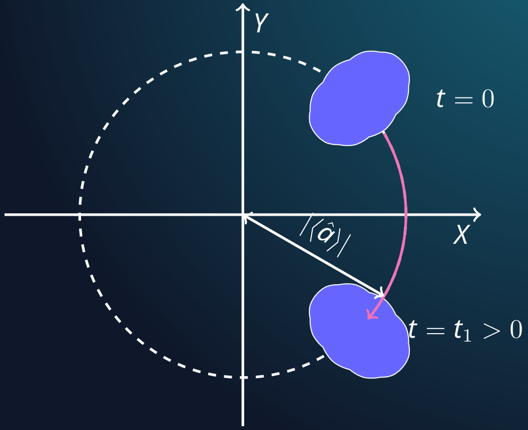
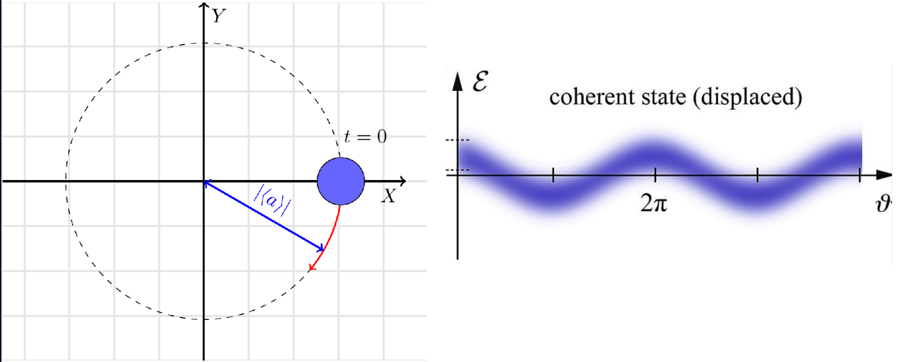


Phase space trajectory

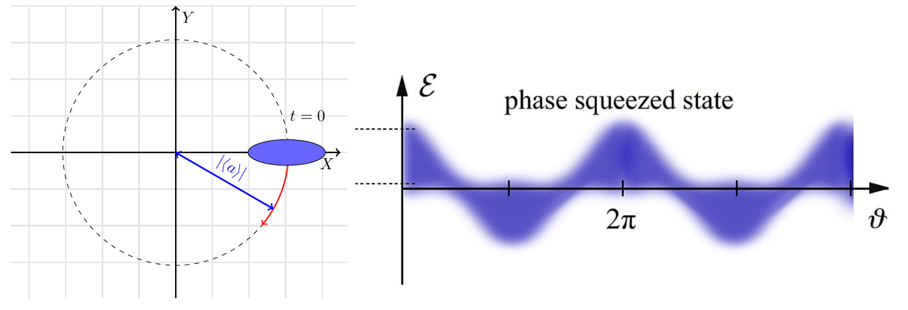
Dynamics

Coherent state



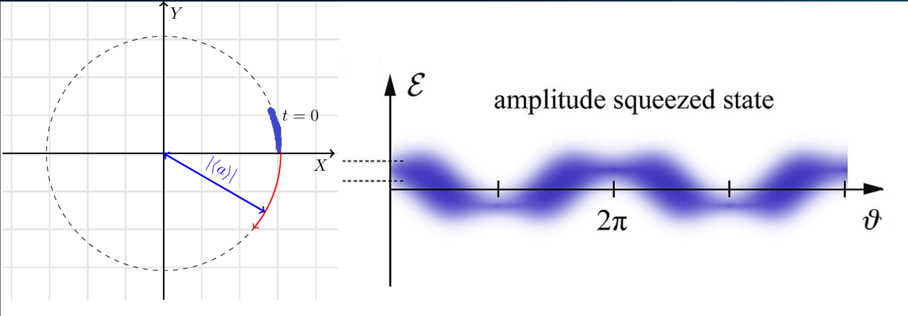
Dynamics

Phase squeezed state



Dynamics

Amplitude squeezed state



Dynamics

Sketch the dynamics of
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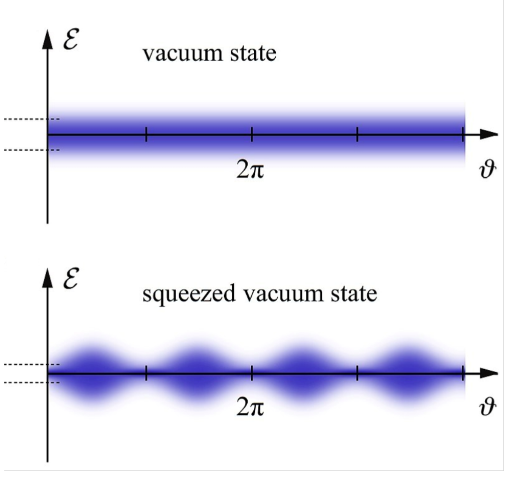

Generation

Recall

Generation of a coherent state

Displacement operators $D(\alpha) = e^{\alpha a^\dagger - \alpha^* a}$ and $|\alpha\rangle = D(\alpha)|0\rangle$.
For an oscillating current source $\mathbf{J} = \mathbf{J}_0(\mathbf{r})e^{i\omega t}$, the interaction Hamiltonian becomes

$$\mathcal{H}_I = \left(V_I a + V_I^* a^\dagger\right),$$

where the interaction potenital is time-indepdenent and reads

$$V_I = i\omega \int dv \mathcal{E}_\omega(\mathbf{r}) \cdot \mathbf{J}_0(\mathbf{r}).$$

The evolution of a state is given by

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$$|\psi(t)\rangle_I = e^{\alpha^* a - \alpha a^\dagger} |0\rangle \qquad \alpha = \frac{iV_I^* t}{\hbar}$$

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Operator

A squeezed state is generated by a squeeze operator,

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Because of a^2 and $(a^\dagger)^2$, we need terms $\mathbf{E}^2$. **Squeezing involves nonlinear process!**

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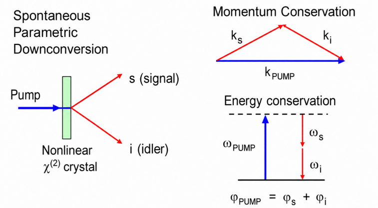
Spontaneous parametric down-conversion



Nonlinear Optics

In a non-centrosymmetric crystal like $LiNbO_3$ or BBO, the optical polarization can have both the linear and quadratic terms,

$$P_i = \chi_{ij}^{(1)} E_j + \chi_{ijk}^{(2)} E_j E_k.$$

If both $E_j \sim e^{-i\omega t}$ and $E_k \sim e^{-i\omega t}$, the output P_i is $\sim e^{-i2\omega t}$

Nonlinear interaction

In materials with χ^2, the ω mode and the 2ω mode can interact!

- second harmonics generation: two ω photons become one 2ω photon
- Parametric down conversion: one 2ω photon becomes two ω photons

Generation: Parametric Down-Conversion

$$\mathcal{H} = \hbar\omega a^\dagger a + \hbar\omega_p \omega a_p^\dagger a_p + i\hbar\beta\chi^{(2)}\left(a^2 a_p^\dagger - (a^\dagger)^2 a_p\right)$$

$\chi^{(2)}$: second order susceptibility
β: parameter related to the modes (overlapping)

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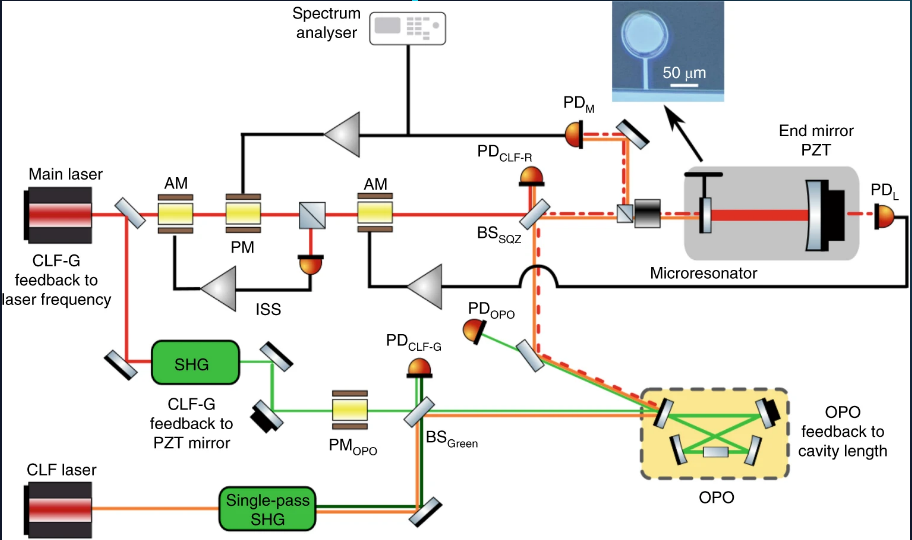
Evolution operator is a squeeze operator!

$$U(t) = \exp\left[\eta^* t a^2 - \eta t (a^\dagger)^2\right] = S(\xi)$$

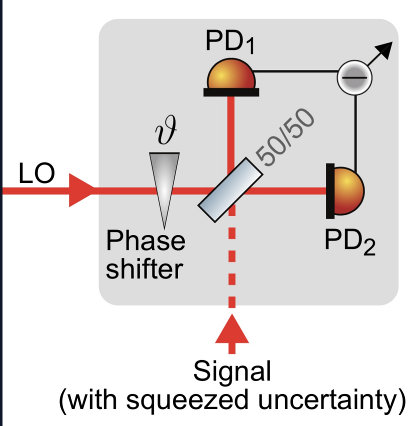
Generation: OPO and Experiment



Nat. Photonics 14, 19–23 (2020).

Detection

balanced homodyne detector (BHD)
Local oscillator (LO): coherent state $|\alpha\rangle$

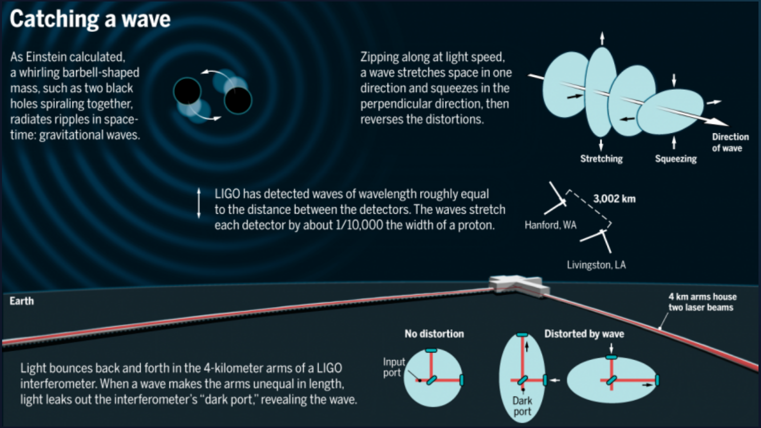


$$I_1 - I_2 = 2|\alpha|\langle X(\theta)\rangle$$

See Sec. 7.3 of C. Gerry and P. Knight

Gravitational Wave Detection

- General relativity: mass *m* distorts space-time; length varies
- Collision of giant masses (black holes) generates gravitational waves

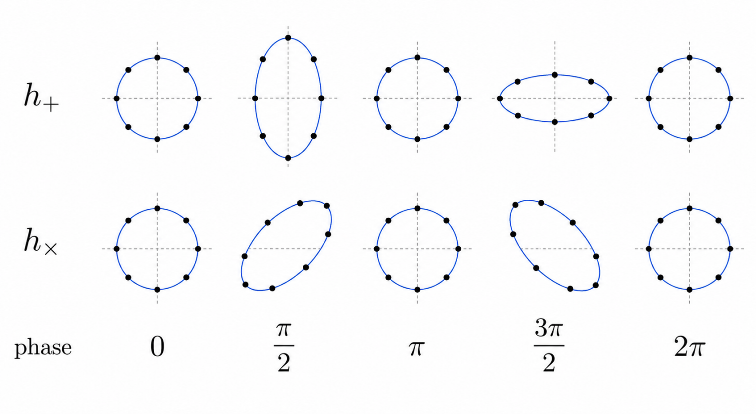


V. ALTOUNIAN/SCIENCE

Gravitational Wave Detection

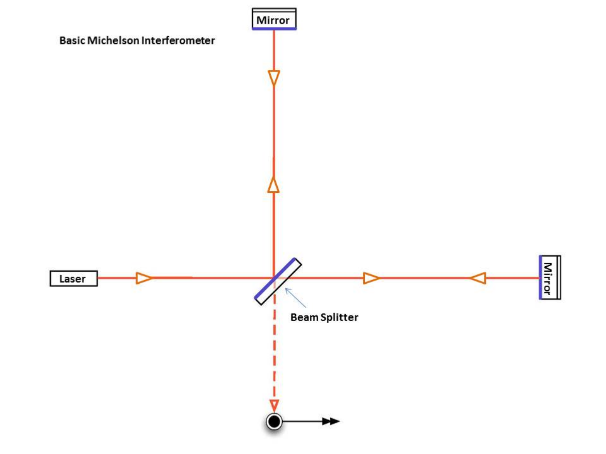
Effects require relative precision: 10^{-18}

Arm length $= 4$ km $\Rightarrow \Delta L = 4 \times 10^{-15}$ m $\sim$ proton size

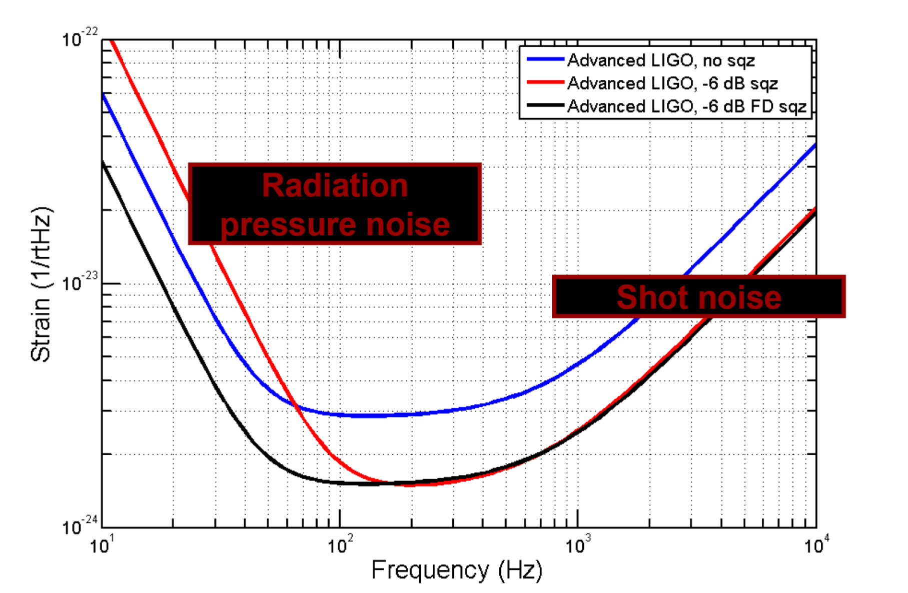


Michaelson Interferometer

Long distance: bigger phase due to gravitational waves Large power: reducing phase uncertainty, but bigger radiation noise.

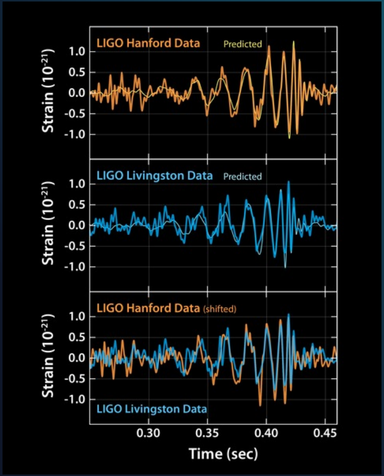


Noise Reduction with Squeezed Lights



Application: Gravitational Wave Detection

- **LIGO Sensitivity:** Requires detecting strain $\sim 10^{-18}m$.
- **Quantum Limit:** Shot noise limits sensitivity at high frequencies.
- **Squeezed Light Solution:**



Quantum Enhancement

Injecting Squeezed Vacuum into the dark port reduces phase uncertainty below the standard quantum limit.

$$\Delta I \propto 2|\alpha| \left\langle \hat{X}(\theta) \right\rangle$$