Non-Classical Light

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Outline

Squeezed Light

- Dynamics
- Generation and Detection
- Application: Gravitation Wave

Dynamics

Motion in the quadrature plane
Light of a single frequency ω,

$$a(t) = a(0)e^{-i\omega t}$$
$$a^\dagger(t) = a^\dagger(0)e^{i\omega t}$$

The evolutions of the quadrature operators are

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$$X(t) = X(0)\cos(\omega t + \phi)$$
$$Y(t) = Y(0)\sin(-\omega t + \phi)$$

Clockwise motion!

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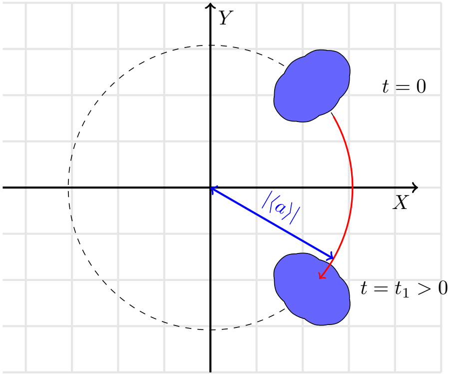

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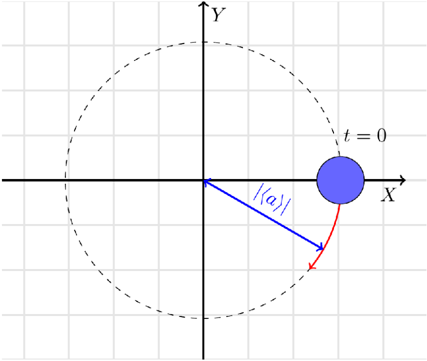
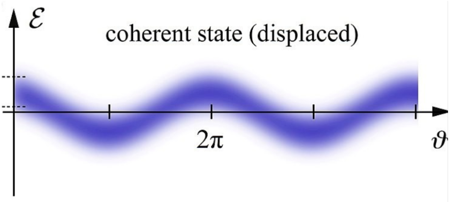
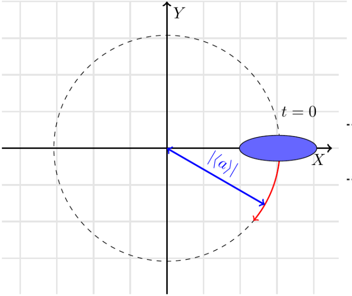
Clockwise motion!

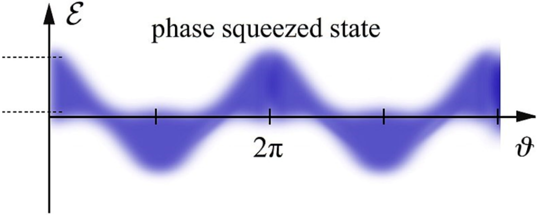
Coherent state



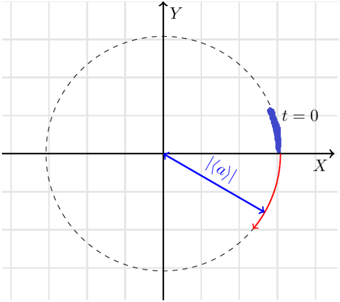


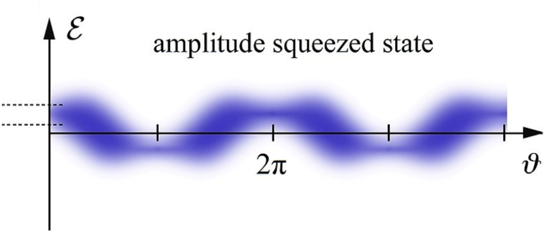
Phase squeezed state





Amplitude squeezed state





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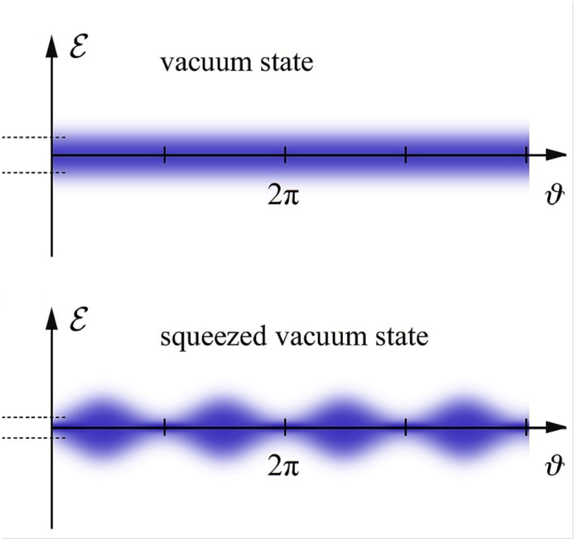

Generation

Recall

Generation of a coherent state

Displacement operators $D(\alpha) = e^{\alpha a^\dagger - \alpha^* a}$ and $|\alpha\rangle = D(\alpha)|0\rangle$.
For an oscillating current source $\mathbf{J} = \mathbf{J}_0(\mathbf{r})e^{i\omega t}$, the interaction Hamiltonian becomes

$$\mathcal{H}_I = \left(V_I a + V_I^* a^\dagger \right),$$

where the interaction potenital is time-indepdenent and reads

$$V_I = i\omega \int dv \boldsymbol{\mathcal{E}}_\omega(\mathbf{r}) \cdot \mathbf{J}_0(\mathbf{r}).$$

The evolution of a state is given by

$$|\psi(t)\rangle_I = e^{\alpha^* a - \alpha a^\dagger}|0\rangle$$

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A squeezed state is generated by a squeeze operator,

$$S(\xi) = \exp\left(\frac{\xi^* a^2 - \xi(a^\dagger)^2}{2}\right).$$

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Because of a^2 and $(a^\dagger)^2$, we need terms of $\mathbf{E}^2$. **Squeezing involves nonlinear process!**

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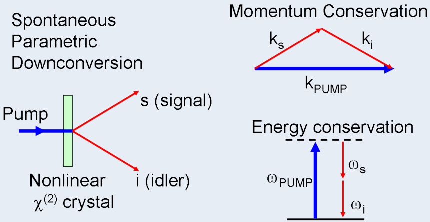
Spontaneous parametric down-conversion



Nonlinear Optics

In a non-centrosymmetric crystal like $LiNbO_3$ or BBO, the optical polarization can have both the linear and quadratic terms,

$$P_i = \chi_{ij}^{(1)} E_j + \chi_{ijk}^{(2)} E_j E_k.$$

If both $E_j \sim e^{-i\omega t}$ and $E_k \sim e^{-i\omega t}$, the output P_i is $\sim e^{-i2\omega t}$

Nonlinear interaction

In materials with χ^2, the ω mode and the 2ω mode can interact!

- second harmonics generation: two ω photons become one 2ω photon
- Parametric down conversion: one 2ω photon becomes two ω photons

Generation: Parametric Down-Conversion

$$\mathcal{H} = \hbar\omega a^\dagger a + \hbar\omega_p\omega a_p^\dagger a_p + i\hbar\beta\chi^{(2)}\left(a^2 a_p^\dagger - (a^\dagger)^2 a_p\right)$$

$\chi^{(2)}$: second order susceptibility
β: parameter related to the modes (overlapping)

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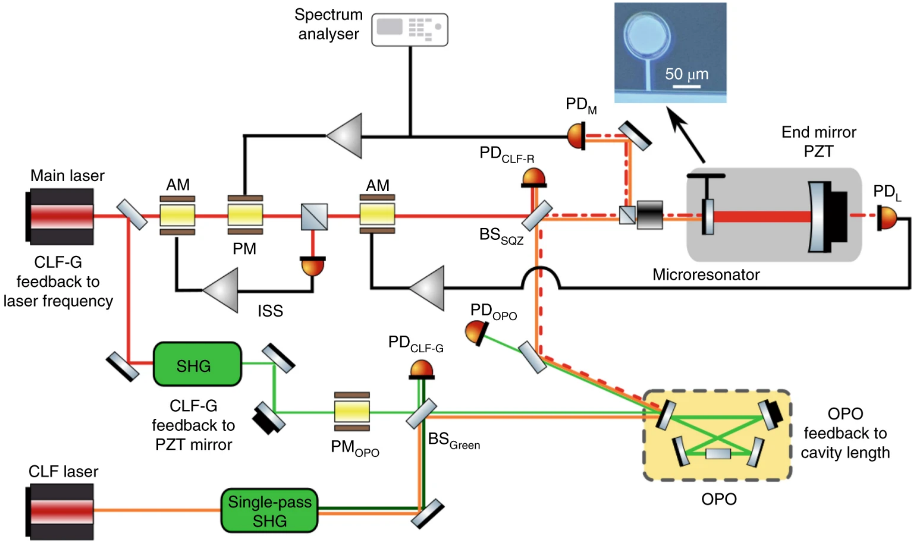
Evolution operator is a squeeze operator!

$$U(t) = \exp\left[\eta^* t a^2 - \eta t(a^\dagger)^2\right] = S(\xi)$$

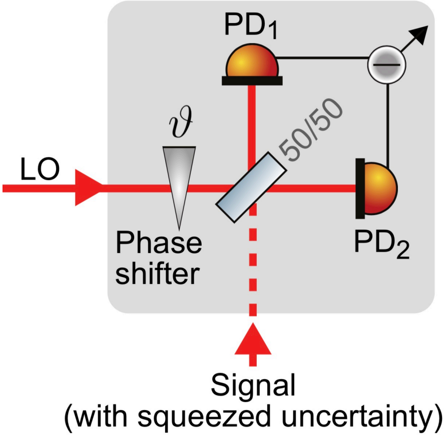
Generation: OPO and Experiment



Nat. Photonics 14, 19–23 (2020).

Detection

balanced homodyne detector (BHD)
Local oscillator (LO): coherent state $|\alpha\rangle$

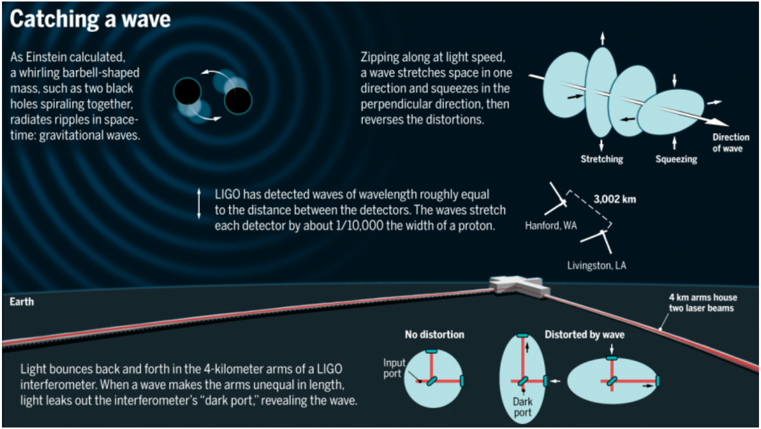


$$I_1 - I_2 = 2|\alpha|\langle X(\theta)\rangle$$

See Sec. 7.3 of C. Gerry and P. Knight

Gravitational Wave Detection

- General relativity: mass m distorts space-time; length varies
- Collision of giant masses (black holes) generates gravitational waves



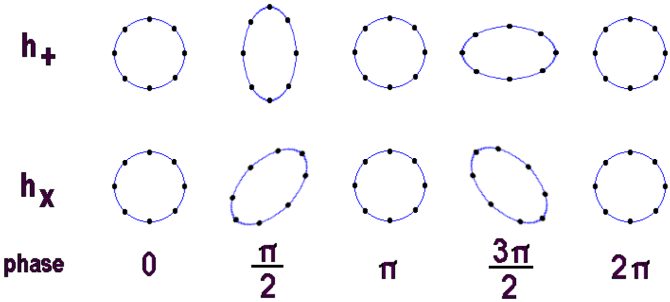
V. ALTOUNIAN/SCIENCE

Gravitational Wave Detection

Effects

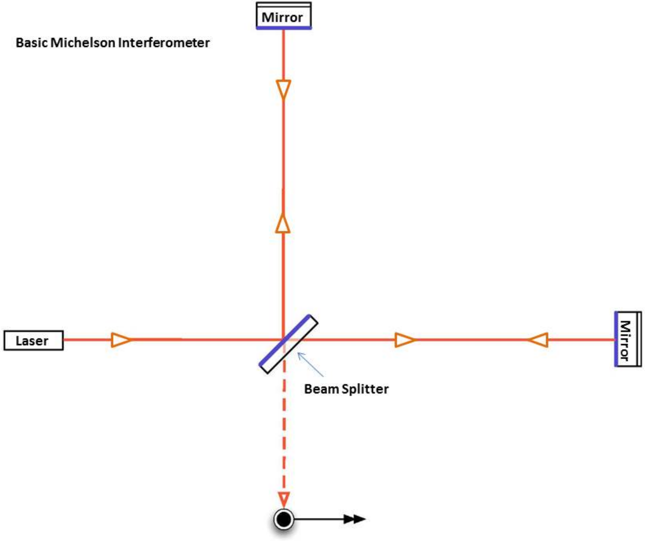
Require relative precision: 10^{-18}

Arm length $= 4$ km $\Rightarrow \Delta L = 4 \times 10^{-15}$m $\sim$ proton size



Michaelson Interferometer

Long distance: bigger phase due to gravitational waves Large power: reducing phase uncertainty; but bigger radiation noise.



Noise Reduction with Squeezed Lights