Photon Statistics

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Outline

Photon-Counting Statistics

- Poisson distributions and coherent lights
- Super-Poisson distributions
- Sub-Poisson distributions

Photon Detection

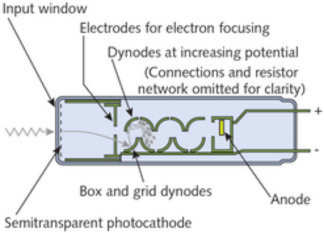
- Theories
- Shot noises
- Observation

Photon-Counting: Detectors

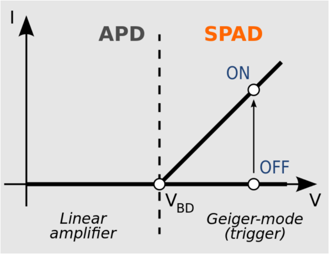
Sensitive light detectors:

- photomultiplier tube (PMT)
- single-photon avalanche diode (SPAD), avalanche photodiode (APD)
- superconducting nanowire single-photon detectors (SNSPDs)

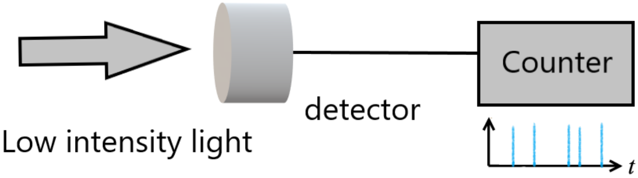
PMT

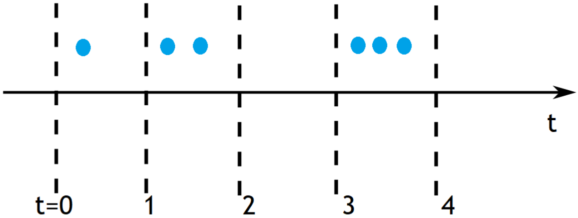


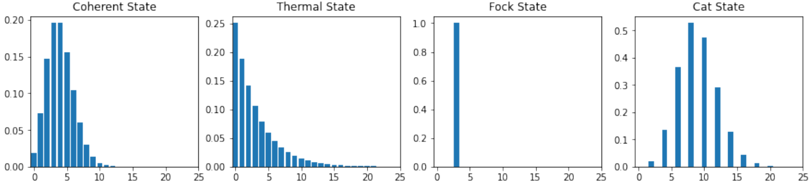
SPAD



Photon-Counting







Photon-Counting Statistics

Photon flux Φ(number of photon per unit time)

$$\Phi = \frac{IA}{\hbar\omega} = \frac{P}{\hbar\omega}$$

Quantum Efficiency η(typical $\sim$ 10 $\%$)

$$\eta = \frac{\text{number of counts}}{\text{number of photons}} = \frac{N(T)}{\Phi T}$$

$N(T)$ is the count number for a given duration T
Counting rate R (upper limit 10^6 counts/sec, dead time $\sim$ 1 μs)

$$R = \frac{N}{T} = \eta\Phi = \frac{\eta P}{\hbar\omega}$$

Photon-Counting Statistics

Maximum Power

Let the maximum counting rate $R = 10^6$ count/sec and $\eta = 15\%$. The photon energy $\hbar\omega$ is 2 eV. What is the maximum power that the photodetector can detect?

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Answer

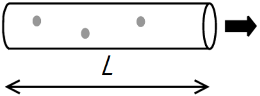
$$R = \frac{\eta P}{\hbar\omega}$$
$$\Rightarrow P = \frac{R\hbar\omega}{\eta} = 2.1 \times 10^{-12} \text{ Watt}$$

Poisson Distribution: Coherent Light

Counting within δt

$L = c\delta t$

$\bar{n} = \frac{\Phi L}{c} = \Phi \delta t$



Probability p_n: find n photons in the cube of a length L. If for a random process, the distribution is a Poisson distribution,

$$P(n) = \frac{\bar{n}^n}{n!} \exp^{-\bar{n}}.$$

Poisson Distribution

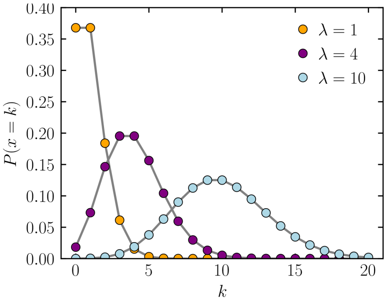
When the occurence of events are independent, the number of the events satisfies a Poisson distribution.

A discrete random varible X with a Poisson Distrution:

$$p(X) = \frac{\lambda^X}{X!} \exp(-\lambda)$$

Average number $\langle X \rangle$ is λ.
Variance $\langle X^2 \rangle - \langle X \rangle^2$ is also λ.

When λ is large, Poisson distribution is close to a Gaussian distribution.



Events

- Calls per hour at a call center
- Typhoons per year
- Number of laser photons hitting a detector in a particular time interval
- Shot noises

Example

Number of Typhoons

There are 5 typhoons in average coming to Taiwan every years.

- What is the probability of 4 typhoons coming to Taiwan next year?
- What is the probability of less than 2 typhoons coming to Taiwan next year?

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Answer

$p(n) = \frac{5^n}{n!} e^{-5}$.

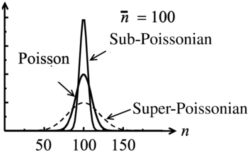
- $p(4) = \frac{5^4}{4!} e^{-5} = 0.175$
- $p = p(0) + p(1) = \frac{5^0}{0!} e^{-5} + \frac{5^1}{1!} e^{-5} = 0.04$

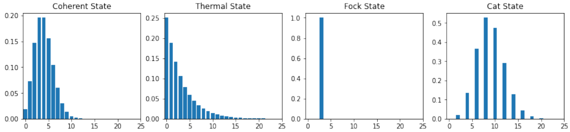
Classification by Photon Counting

(1) Super-Poissonian $\sigma(n) > \sqrt{\bar{n}}$: thermal, chaotic lights, mixed ensembles, non-classical light (?)

(2) Poissonian Light $\sigma(n) = \sqrt{\bar{n}}$: coherent lights

(3) Sub-Poissonian Light $\sigma(n) < \sqrt{\bar{n}}$: non-classical light



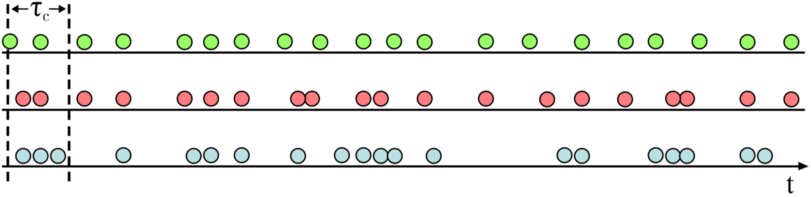


Photon Bunching and Antibunching

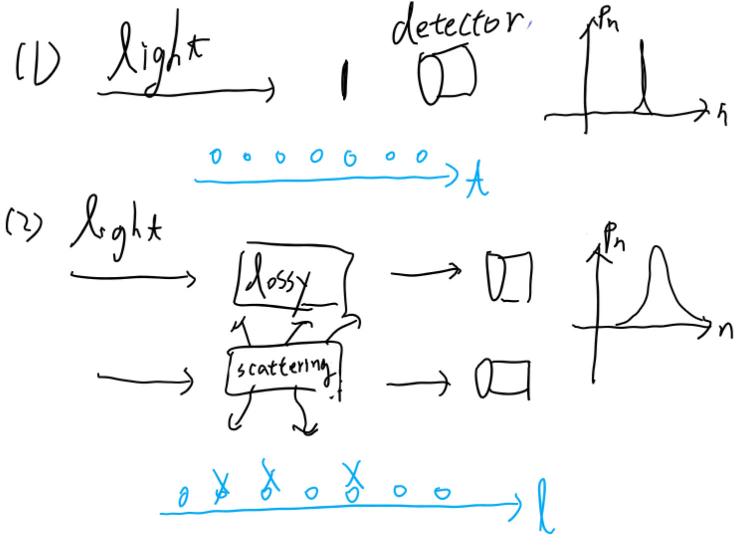
Photon counting and beyond

Photon counting: information of n

Correlation: statistical relationship between photons. Coherence functions (correlation function) g_1, g_2, ... Antibunching (single photon), Random (laser), Bunching (thermal)



Degradation of Photon Statistics



Photon Detection

- Classical theory: treat $I(t)$ as a continuous number
  - constant intensity $I(t) = I_0 \Rightarrow$ Poisson distribution $\sigma(n) = \sqrt{\bar{n}}$
  - time-dependent $I(t) \Rightarrow$ super-Poisson distribution $\sigma(n) > \sqrt{\bar{n}}$
- Quantum theory

$$\bar{N} = \eta\bar{n}$$
$$\sigma^2(N) = \eta^2\sigma^2(n) + \eta(1-\eta)\bar{n}$$

(1) If $\eta = 1$, statistics of N is the same as n
(2) If $\eta \ll 1$, $\sigma(N) = \sqrt{\bar{N}} \Rightarrow$ always Poissonian N regardless n.

High quantum efficiency is important for sub-Poissonian photodetection.

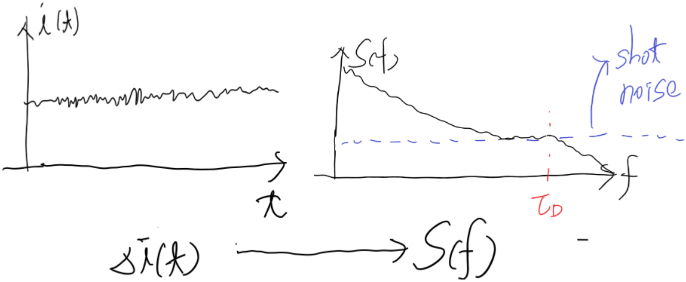
Shot Noise

Detection with photodiode (PD): high-intensity light beam hits on PD and generate a current $i(t)$. Let $i(t) = \langle i \rangle + \Delta i(t)$. Because of the particle-like property, the noise $\Delta i(t)$ has a flat spectrum (white noise). If the detector has a band width Δf,

$$\langle \Delta i^2 \rangle = 2e\Delta f \langle i \rangle,$$

Noise power is

$$P_{\text{noise}} = R_L \langle \Delta i^2 \rangle = 2e\Delta f \langle i \rangle,$$



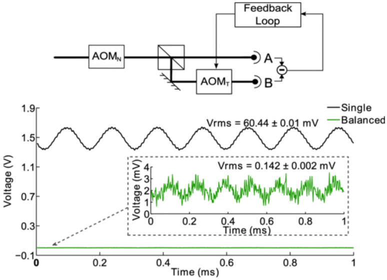
Shot Noise

Classical Noise

- Continuous fluctuation
- noise power spectrum is large at low frequencies
- can be eliminated by beam splitter techniques

Shot Noise

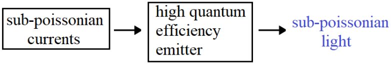
- Due to the discrete nature
- $\Delta i(t)$ is full random
- noise power spectrum is flat
- can not be eliminated

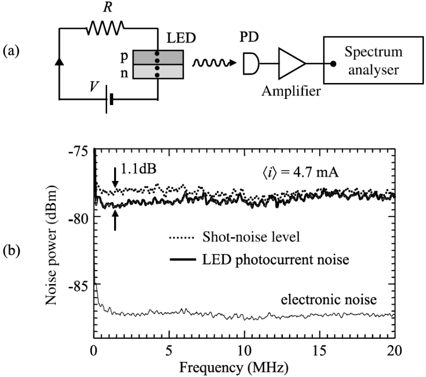


Review of Scientific Instruments 83, 063705 (2012)

Generation of Sub-Poissonian Light

- Sub-Poissonian currents to lights, for examples, PMTs, LEDs of high quantum efficiency η
- Squeezed light by nonlinear optics





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