Quantum Information in A Nutshell

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Outline

Quantum Information

- Types of Quantum Technologies
- Qubits and Gates
- Entanglement
- Algorithms

Reference:Physics 160 Lecture Notes by Prof. Mikhail Lukin

Types of Quantum Technologies

Classification by Prof. Lukin

- **Quantum Metrology**: superposition and entanglement to make more precise measurements
- **Quantum Communication**: superposition and entanglement to transmit information in a secure way. **No-cloning theorem!**
- **Quantum Computing**: a superposition of input states, different inputs interfere, provides "**quantum parallelism**". Quantum Fourier Transform.

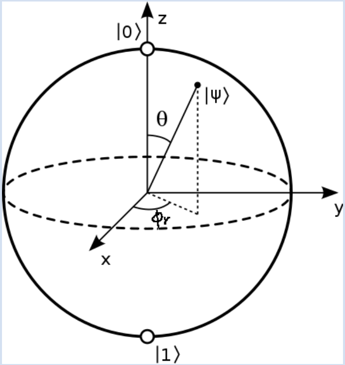
Qubits and Bloch Sphere

A qubit is a two-state system:

$$|Q\rangle = c_0|0\rangle + c_1|1\rangle.$$

$|0\rangle$ and $|1\rangle$ are arbitrary two different states. For example, they can be $|H\rangle$ and $|V\rangle$ of a plane wave.

$$c_0 = \cos\frac{\theta}{2} \quad c_1 = \sin\frac{\theta}{2} e^{i\phi}$$



Single Qubit Operations (Gates)

Operations

Single qubit operations are rotations on the Blcoh sphere.

X gate

180°-Rotation about the x-axis

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$X \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Arbitrary-angle $\Delta\theta$-rotation

$$R_X(\Delta\theta) = e^{-i\frac{\Delta\theta}{2}X}$$

Single Qubit Operations (Gates)

Operations

Single qubit operations are rotations on the Blcoh sphere.

Y gate

180°-Rotation about the y-axis

$$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$Y \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} = i \begin{pmatrix} \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

Arbitrary-angle $\Delta\theta$-rotation

$$R_Y(\Delta\theta) = e^{-i\frac{\Delta\theta}{2}Y}$$

Single Qubit Operations (Gates)

Operations

Single qubit operations are rotations on the Blcoh sphere.

Z gate

180°-Rotation about the z-axis

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$Z \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix}$$

Arbitrary-angle $\Delta\theta$-rotation

$$R_z(\Delta\theta) = e^{-i\frac{\Delta\theta}{2}Z}$$

Single Qubit Operations (Gates)

Hadamard gate

180°-Rotation about the $\left(\frac{\hat{x}+\hat{z}}{\sqrt{2}}\right)$-axis

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$$

Question

$$H|X,\uparrow\rangle = ?$$

Single Qubit Operations (Gates)

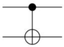
Operator	Gate(s)		Matrix
Pauli-X (X)	X	$\oplus$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Pauli-Y (Y)	Y		$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$
Pauli-Z (Z)	Z		$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Hadamard (H)	H		$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
Phase (S, P)	S		$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$
$\pi/8$ (T)	T		$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$

Two-qubit Operations (Gates)

Controlled NOT gate (Controlled X gate)

If the first qubit is $|0\rangle$, do nothing on the second qubit.
If the first qubit is $|1\rangle$, apply X on the second qubit.

Controlled Not (CNOT, CX)

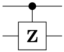


$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{matrix} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{matrix}$$

Controlled Z gate

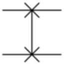
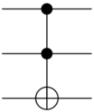
If the first qubit is $|0\rangle$, do nothing on the second qubit.
If the first qubit is $|1\rangle$, apply Z on the second qubit.

Controlled Z (CZ)



$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

More Gates

SWAP			$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
Toffoli (CCNOT, CCX, TOFF)			$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

Universal Set of Quantum Gates

Universality Theorem

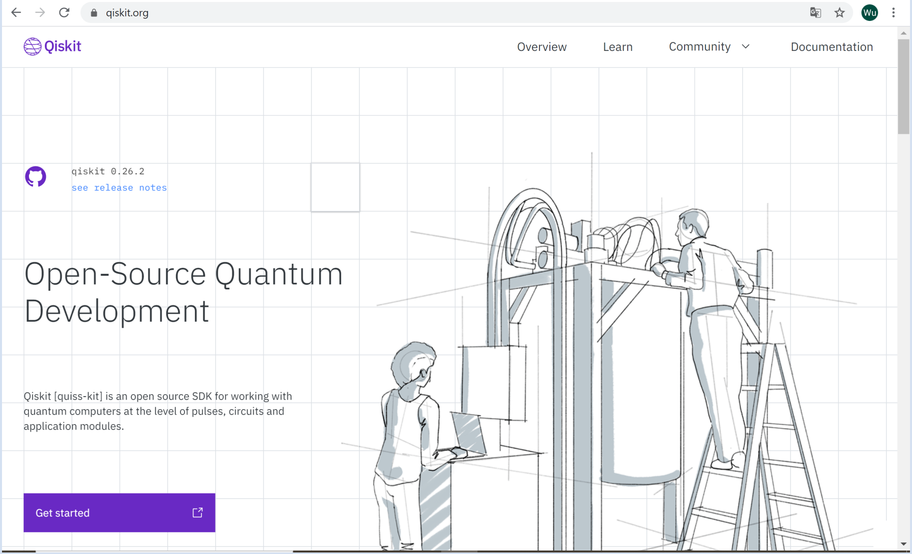
Any n-dimensional unitary operator can be decomposed as a product of two-dimensional operators.

Specific Sets of Quantum Gates

1. H, CNOT, and T
2. Toffoli and H

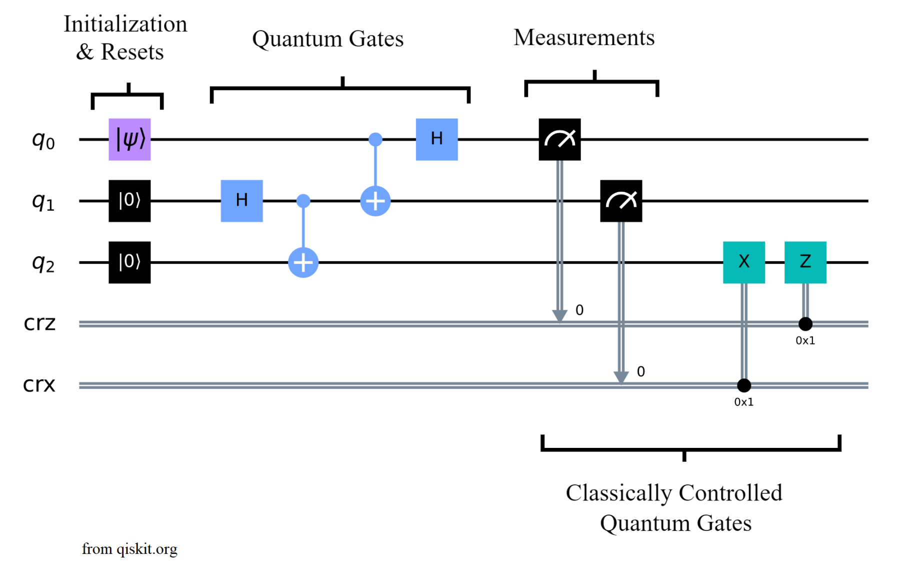
Quantum Programing

IBM Qiskit



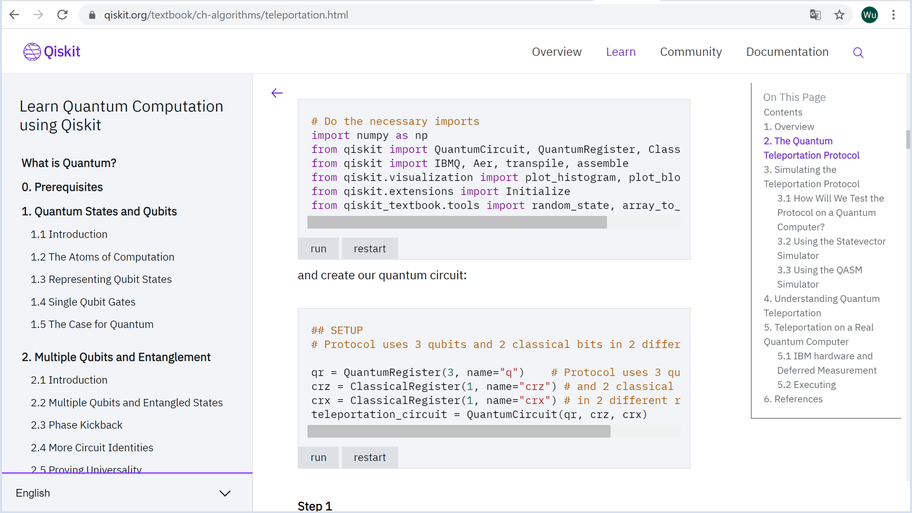
Quantum Programing

Quantum circuit



Quantum Programing

You can run the quantum code with python and real qubits!



qiskit.org/textbook/ch-algorithms/teleportation.html

Qiskit   Overview   Learn   Community   Documentation

Learn Quantum Computation using Qiskit

English

```
# Do the necessary imports
import numpy as np
from qiskit import QuantumCircuit, QuantumRegister, Class
from qiskit import IBMQ, Aer, transpile, assemble
from qiskit.visualization import plot_histogram, plot_blo
from qiskit.extensions import Initialize
from qiskit_textbook.tools import random_state, array_to_
```

run   restart

and create our quantum circuit:

```
## SETUP
# Protocol uses 3 qubits and 2 classical bits in 2 differ

qr = QuantumRegister(3, name="q")    # Protocol uses 3 qu
crz = ClassicalRegister(1, name="crz") # and 2 classical
crx = ClassicalRegister(1, name="crx") # in 2 different r
teleportation_circuit = QuantumCircuit(qr, crz, crx)
```

run   restart

Step 1

On This Page

Bell States

Definition

A state is entangled if

$$|\psi\rangle \neq |\psi\rangle_A \otimes |\psi\rangle_B.$$

Bell States

Two-level systems A and B. Bell states are

$$|\Phi^{\pm}\rangle_{AB} = \frac{|00\rangle \pm |11\rangle}{\sqrt{2}}$$
$$|\Psi^{\pm}\rangle_{AB} = \frac{|01\rangle \pm |10\rangle}{\sqrt{2}}$$

Measurement by Alice affects the reduced density matrix of Bob!

Einstein–Podolsky–Rosen Paradox

EPR Paradox

Einstein did not believe that measurement by Alice can affect the spatially separate state of Bob. His thought was local reality. In stead of 100% in

$$\frac{|01\rangle + |10\rangle}{\sqrt{2}},$$

he thought that 50% in

$$|10\rangle$$

and 50% in

$$|01\rangle$$

He thought the state was not superposition! There was a reality before measurement.

Bell Inequality

To determine whether

100% in

$$\frac{|01\rangle + |10\rangle}{\sqrt{2}},$$

50% in

$$|10\rangle$$

50% in

$$|01\rangle$$

John Bell in 1964, if local reality, then

$$|\ C_h(a,b) - C_h(a,c)\ | \leq 1 + C_h(b,c)$$

The inequality basically describes the **correlations**. But, this inequality can be violated if it is the left case.

Experiments showed that the Bell's inequality is violated.

Superposition and entanglement are true!!

Application of Entanglement

- Quantum key distribution
- Superdense coding
- Quantum teleportation

Quantum Algorithms

- Deutsch's Algorithm
  Determine if a function $f(x)$ is constant for $x = 0, 1$ and $f(x) = 0, 1$.
- Deutsch-Jozsa algorithm
  Determine if a function $f(x_n)$ is constant for all $x_n = 0, 1$ and $f(x_n) = 0, 1$.
- Grover's Algorithm
  Find x_i such $f(x_i) = 1$ when all other $f(x_n) = 0$. If the number of x_n is N, the classical algorithm takes $O(N)$ steps. Grover's algorithm takes $O(\sqrt{N})$ steps.
- Shor's Algorithm
  Decompose a product of two large prime number $N = pq$.
  Acceleration from exponential time to polynomial time.